

Fractions, Decimals & Percentages



Years 4, 5 and 6

A Guide for Parents

At North Park Primary, we believe that children should be confident and proficient mathematicians. We have a 'Can do' attitude towards maths and the support of parents in developing this is crucial. When working together as a partnership, parents and school can foster an enthusiasm in maths to support children in their mathematical self-belief. At North Park Primary we follow the White Rose Maths Hub schemes of learning.

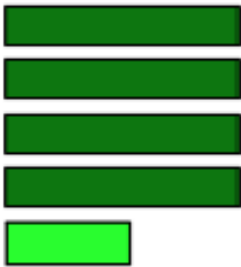
When planning lessons, teachers follow the cycle of 'concrete', pictorial, abstract' (CPA approach) and this guidance aims to set out examples of how we develop children's skills of calculating with fractions, decimals and percentages using this cycle of teaching.

'Concrete'- Each skill is often first modelled with concrete materials (e.g. base ten, cubes, cuisenaire rods). This is the "doing stage". During this stage, students use concrete objects to model problems. The CPA approach brings concepts to life by allowing children to experience and handle physical (concrete) objects. For example, if a problem involves adding pieces of fruit, children can use counters or cubes which represent the fruit.

'Pictorial'- Pictorial is the "seeing" stage. Here, visual representations of concrete objects are used to model problems. This stage encourages children to make a mental connection between the physical object they just handled and the abstract pictures, diagrams or models that represent the objects from the problem.

'Abstract'- Abstract is the "symbolic" stage, where children use abstract symbols to model problems. Students will not progress to this stage until they have demonstrated that they have a solid understanding of the concrete and pictorial stages of the problem. The abstract stage involves the teacher introducing abstract concepts (for example, mathematical symbols). Children are introduced to the concept at a symbolic level, using only numbers, notation, and mathematical symbols (for example, +, −, x, /) to indicate addition, multiplication or division.

Fractions

Year 4	Year 5	Year 6
<p>Recognise and show, using diagrams, families of common equivalent fractions.</p> <p>Add and subtract fractions with the same denominator.</p> <p>Calculate fractions of a quantity.</p>	<p>Compare and order fractions whose denominators are all multiples of the same number.</p> <p>Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths.</p> <p>Convert mixed and improper fractions.</p> <p>Add and subtract fractions with the same denominator and denominators that are multiples of the same number.</p> <p>Multiply proper fractions and mixed numbers by whole numbers.</p> <p>Fractions of an amount.</p>	<p>Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions.</p> <p>Multiply simple pairs of proper fractions, writing the answer in its simplest form.</p> <p>Divide proper fractions by whole numbers.</p> <p>Use common factors to simplify fractions.</p> <p>Compare and order fractions including fractions >1.</p>
<p><u>Recognise and show, using diagrams, families of common equivalent fractions.</u></p> <p>Concrete-Children will use strip diagrams to investigate and record equivalent fractions. They will start to see links between the numerator and the denominator.</p> <p>Use two strips of equal sized paper.</p> <p>Fold one strip into quarters and the other into eighths.</p> <p>Place the quarters on top of the eighths and lift up one quarter; how many eighths can you see?</p> <p>How many eighths are equivalent to one quarter?</p> <p>Which other equivalent fractions can you find?</p> <p>Pictorial- Children will use fraction walls and bar models to explore equivalent fractions further. Attention should be drawn to the method of multiplying the numerators and denominators by the same number to ensure that fractions are equivalent.</p>	<p><u>Compare and order fractions whose denominators are all multiples of the same number.</u></p> <p>Concrete- Use concrete resources where necessary e.g. cubes.</p> <p>Pictorial- Children can use bar models to support their understanding. Children will compare and order the fractions by finding a common denominator. They will do this by finding the lowest common multiple of both denominators. Whatever they have multiplied the original denominators by to get the common one, they need to multiply the numerator by too. Once they have done this, they will only need to look at the numerator to decipher which is bigger.</p>	<p><u>Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions.</u></p> <p>Concrete-Practical resources such as cuisenaire allow you to see mixed fractions:</p>  <p>If dark green is one whole then...</p> <p>What is light green? What is the total of this number?</p> <p>What if I had 2 more wholes? What would I have in total now?</p> <p>Pictorial-</p>

How many fractions that are equivalent to one half can you see on the fraction wall?



Draw extra rows to show other equivalent fractions.

Using the diagram, complete the equivalent fractions.



$$\frac{1}{4} = \frac{\square}{12} \quad \frac{1}{\square} = \frac{6}{12} \quad \frac{2}{3} = \frac{\square}{12} \quad \frac{5}{12} = \frac{\square}{24}$$

Abstract-

Complete:

$$\frac{1}{4} = \frac{2}{\square} = \frac{\square}{12} = \frac{4}{\square} = \frac{\square}{100} = \frac{\square}{500}$$

Non-statutory- Pupils should simplify using factors and multiples where appropriate (for example, $6/9 = 2/3$ or $1/4 = 2/8$.) Children should use the method of finding the highest common factor of both the numerator and the denominator and divide both by this.

Adding fractions with the same denominator

Concrete-Children will use practical equipment to add fractions. Children will record their answer as an improper fraction when the total is more than 1.

Take two identical strips of paper.

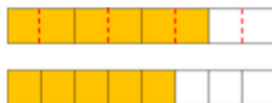
Fold your paper into quarters.

Can you use the strips to solve

$$\frac{1}{4} + \frac{1}{4} \quad \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \quad \frac{3}{4} + \frac{3}{4} \quad \frac{\square}{4} + \frac{\square}{4} = \frac{7}{4}$$

What other fractions can you make and add?

Use bar models to compare $\frac{5}{8}$ and $\frac{3}{4}$



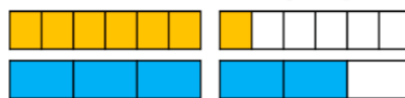
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Use this method to help you compare:

$$\frac{5}{6} \text{ and } \frac{2}{3} \quad \frac{2}{3} \text{ and } \frac{5}{9} \quad \frac{7}{16} \text{ and } \frac{3}{8}$$

Use bar models to compare $\frac{7}{6}$ and $\frac{5}{3}$



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Use this method to help you compare:

$$\frac{5}{2} \text{ and } \frac{9}{4} \quad \frac{11}{6} \text{ and } \frac{5}{3} \quad \frac{9}{4} \text{ and } \frac{17}{8}$$

Abstract-

Order the fractions from greatest to smallest:

$$\frac{3}{7}, \frac{3}{5} \text{ and } \frac{3}{8} \quad \frac{2}{3}, \frac{5}{6} \text{ and } \frac{7}{12} \quad \frac{6}{11}, \frac{3}{5} \text{ and } \frac{2}{3}$$

Order the fractions from greatest to smallest using common denominators:

$$\frac{8}{5}, \frac{11}{10} \text{ and } \frac{17}{20} \quad \frac{?}{20}, \frac{?}{20} \text{ and } \frac{?}{20} \quad 1\frac{2}{3}, 1\frac{7}{24} \text{ and } \frac{11}{12}$$

Identify, name and write equivalent fractions of a given fraction

Concrete- Children will explore equivalent fractions using models and concrete representations.

If we take the example of adding 2 and $2/3$ with 3 and $3/4$ then we can consider that the wholes and parts can be combined separately before being combined as two new totals.



So we have:



5 wholes plus a further...

1 whole and $5/12$



Pictorial- A common misconception is to add the denominators as well as the numerators. Use bar models to support children's understanding of why this is incorrect.

Use the models to add the fractions:



Use the number line to add the fractions.



Abstract-

$$\frac{4}{9} + \frac{5}{9} + \frac{8}{9} = \frac{17}{9} \quad \frac{1}{9} + \frac{11}{9} = 1 \quad \frac{17}{9} + \frac{5}{9} + \frac{7}{9} = \frac{29}{9}$$

Subtracting fractions with the same denominator

Concrete- Children use practical equipment and pictorial representations to subtract fractions with the same denominator.

Use identical strips of paper and fold them into eighths.

Use the strips to solve the calculations.

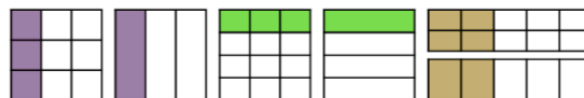
$$\frac{8}{8} - \frac{3}{8} = \frac{5}{8} \quad \frac{7}{8} - \frac{3}{8} = \frac{4}{8} \quad \frac{16}{8} - \frac{9}{8} = \frac{7}{8} \quad \frac{13}{8} - \frac{6}{8} = \frac{7}{8}$$

Pictorial/Abstract- Encourage children to explore subtraction as takeaway and as difference.

Take two pieces of paper the same size.
Fold one piece into two equal pieces.
Fold the other into eight equal pieces.
What equivalent fractions can you find?

Pictorial/Abstract-

Use the models to write equivalent fractions.



Eva uses the models and her multiplication and division skills to find equivalent fractions.



Use this method to find equivalent fractions to $\frac{1}{4}$, $\frac{3}{4}$ and $\frac{4}{4}$ where the denominator is 16

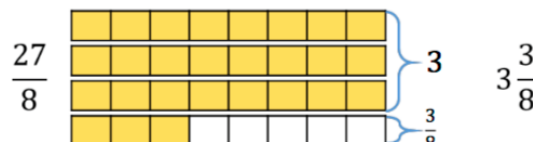
Non-statutory- Pupils connect equivalent fractions to simplifying.

Convert mixed and improper fractions.

Improper to mixed

Children convert improper f

Tommy converts the improper fraction $\frac{27}{8}$ into a mixed number using bar models.



fractions to mixed numbers for the first time.

Concrete-

Combining both of these physically, pictorially or abstract will allow us to see that there are 6 wholes and 5/12.

Abstract-

2 and $\frac{2}{3}$ plus $3 \frac{3}{4}$

2 wholes plus 3 wholes is 5 wholes

This leaves us with $\frac{2}{3}$ plus $\frac{3}{4}$

Now we are simply adding two fractions with different denominators.

The rules of equivalence inform us that the lowest common multiple is 12 and so we convert both fractions.

$$\frac{2}{3} \times 4 = \frac{8}{12} \quad \text{and} \quad \frac{3}{4} \times 3 = \frac{9}{12}$$

Now we can add them together which will total $\frac{17}{12}$ or 1 whole and $\frac{5}{12}$.

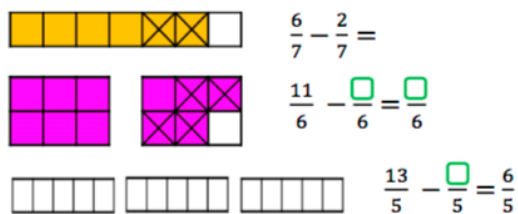
If we combine this with 5 wholes, we would have 6 wholes and $\frac{5}{12}$.

With subtraction, use the same method as Y5.

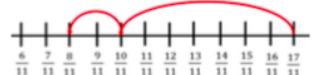
Multiply simple pairs of proper fractions, writing the answer in its simplest form.

Concrete-

Use the bar models to subtract the fractions.



Annie uses the number line to solve $\frac{17}{11} - \frac{9}{11}$



Use a number line to solve:

$$\frac{16}{13} - \frac{9}{13} \quad \frac{16}{9} - \frac{9}{9} \quad \frac{16}{7} - \frac{9}{7} \quad \frac{16}{16} - \frac{9}{16}$$

$$\frac{6}{4} - \frac{3}{4} = \frac{3}{4}$$



Calculate fractions of a quantity.

This is a continuation of what they began to notice in Year 3. Dividing the amount by the denominator and multiplying by the numerator.

Concrete-

Mo has 12 apples.

Use counters to represent his apples and find:

$$\frac{1}{2} \text{ of } 12 \quad \frac{1}{4} \text{ of } 12 \quad \frac{1}{3} \text{ of } 12 \quad \frac{1}{6} \text{ of } 12$$

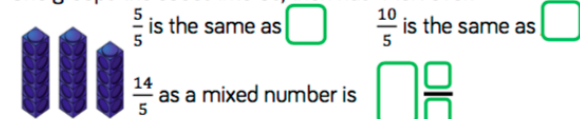
Now calculate:

$$\frac{2}{2} \text{ of } 12 \quad \frac{3}{4} \text{ of } 12 \quad \frac{2}{3} \text{ of } 12 \quad \frac{5}{6} \text{ of } 12$$

What do you notice? What's the same and what's different?

Whitney converts the improper fraction $\frac{14}{5}$ into a mixed number using cubes.

She groups the cubes into 5s, then has 4 left over.



Use Whitney's method to convert $\frac{11}{3}$, $\frac{11}{4}$, $\frac{11}{5}$ and $\frac{11}{6}$

Pictorial-

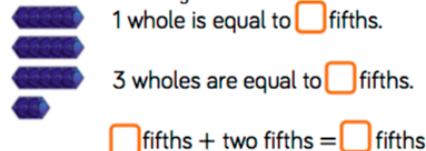
Abstract-

convert $\frac{25}{8}$, $\frac{27}{6}$, $\frac{18}{7}$ and $\frac{32}{4}$

Mixed to improper

Concrete-

Whitney converts $3 \frac{2}{5}$ into an improper fraction using cubes.



Use Whitney's method to convert $2 \frac{2}{3}$, $2 \frac{2}{4}$, $2 \frac{2}{5}$ and $2 \frac{2}{6}$

Pictorial-

Jack uses bar models to convert a mixed number into an improper fraction.



$$2 \frac{3}{5} = 2 \text{ wholes} + 3 \text{ fifths}$$

$$2 \text{ wholes} = 10 \text{ fifths} \\ 10 \text{ fifths} + 3 \text{ fifths} = 13 \text{ fifths}$$

Abstract-

Start with what they do know:



Each pink block is worth $\frac{1}{4}$

so $\frac{1}{4} \times 3 = 3$ groups of $\frac{1}{4}$

We can write this as:

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$



Each pink block is worth $\frac{1}{4}$

so $\frac{1}{4} \times 1 = 1$ group of $\frac{1}{4}$ is $\frac{1}{4}$

We can write this as:

$$\frac{1}{4} = \frac{1}{4}$$



Each pink block is worth $\frac{1}{4}$

So $\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$ of 1 group of $\frac{1}{4}$

We can write this as:

1 group of $\frac{1}{4}$ is $\frac{1}{4}$

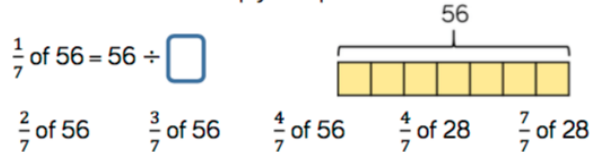
$\frac{1}{3}$ of 1 group of $\frac{1}{4}$ is $\frac{1}{12}$.

First, you split into three equal sections as we need to find $\frac{1}{3}$ of one group of $\frac{1}{4}$.



Pictorial-

Use a bar model to help you represent and find:



Abstract-

Whitney eats $\frac{3}{8}$ of 240 g bar of chocolate.
How many grams of chocolate has she eaten?

convert $2\frac{1}{6}$, $4\frac{1}{6}$, $4\frac{1}{3}$ and $8\frac{2}{3}$

Add and subtract fractions with the same denominator and denominators that are multiples of the same number.

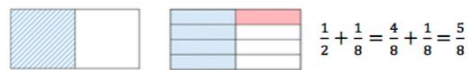
Children add fractions with different denominators for the first time where one denominator is a multiple of the other.

Concrete- Use concrete materials where necessary to recap work from Year 4.

Pictorial- Children use pictorial representations to convert the fractions so they have the same denominator.

Mo is calculating $\frac{1}{2} + \frac{1}{8}$

He uses a diagram to represent the sum.



Use Mo's method to solve:

$$\frac{1}{2} + \frac{3}{8} \quad \frac{1}{4} + \frac{3}{8} \quad \frac{7}{10} + \frac{1}{5}$$

Rosie is using a bar model to solve $\frac{1}{4} + \frac{3}{8}$

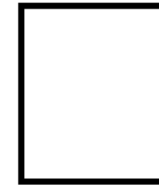


Use a bar model to solve:

$$\frac{1}{6} + \frac{5}{12} \quad \frac{2}{9} + \frac{1}{3} \quad \frac{1}{3} + \frac{4}{15}$$

Asking the pupils to draw a representation either on a bar or a circle can be useful.

Pictorial-



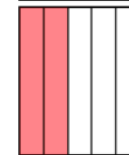
If this is one whole.

$$1 \times 1 = 1$$



Then this is the whole shared equally into 8 equal parts.

This is 3 of the 8 equal parts.
1 group of $\frac{3}{8} = \frac{3}{8}$



Then this is the whole shared equally into 5 equal parts.

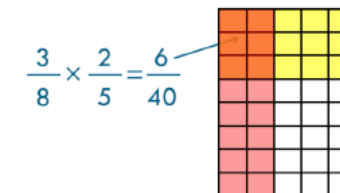
This is 2 of the 5 equal parts.
1 group of $\frac{2}{5} = \frac{2}{5}$

$$\frac{3}{8} \times \frac{2}{5}$$



Relate to what pupils know already.

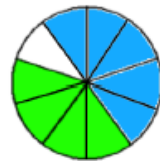
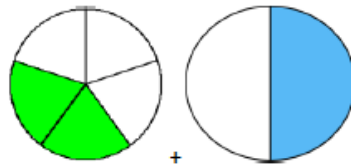
When we had 1 group of $\frac{2}{5}$, the product was $\frac{2}{5}$. Now we are calculating what $\frac{3}{8}$ of 1 group of $\frac{2}{5}$ would be.



6 and 40 have common factors of 1 and 2. Dividing it by the Highest Common Factor (HCF) will provide this fraction in its simplest form: $\frac{3}{20}$

Abstract-

$$2/5 + 1/2$$



Abstract-

When adding two fractions with different denominators, we need to look for the lowest common multiple and convert both numbers (if necessary) to this fraction.

For example, 5 and 2 have the lowest common multiple of 10 – so to add these together, we need to ensure we keep to rules of equivalence (same rule applied to numerator and denominator).

$$\begin{array}{ccc} \begin{array}{c} \text{x2} \curvearrowright \\ \frac{2}{5} \\ \text{x2} \curvearrowleft \\ = \frac{4}{10} \end{array} & + & \begin{array}{c} \text{x5} \curvearrowright \\ \frac{1}{2} \\ \text{x5} \curvearrowleft \\ = \frac{5}{10} \end{array} \\ & + & \end{array}$$

N.B When adding mixed numbers, children will add the wholes and then the fractions. When subtracting mixed numbers where the 2nd fractions is bigger than the 1st, borrow from the whole to create an improper (1st fraction). E.g.

$$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

$$\frac{2}{3} \times \frac{1}{4} = \frac{2}{12} = \frac{1}{6}$$

They should be expected to draw it to continue demonstrating, and developing, their conceptual understanding.

This will support 'genuine' fluency and therefore application.

Divide proper fractions by whole numbers.

Concrete-

Always start with a familiar context.

1/2 of a pizza is to be shared between four friends. How much do they each get?



$$5\frac{5}{6} - 2\frac{1}{3} =$$

$$4\frac{11}{6} - 2\frac{1}{3} =$$


$$4\frac{11}{6} - 2\frac{2}{6} = 2\frac{9}{6} = 3\frac{3}{6} = 3\frac{1}{2}$$

Multiply proper fractions and mixed numbers by whole numbers.

Children are introduced to multiplying fractions by a whole number for the first time. They link this to repeated addition and see that the denominator stays the same.

Pictorial/ Abstract-

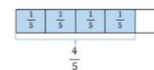
Work out $\frac{1}{6} \times 4$ by counting in sixths.

$$\frac{1}{6} \times 4 = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$


Use this method to work out:

$$2 \times \frac{1}{3} \qquad \frac{1}{5} \times 3 \qquad 6 \times \frac{1}{10}$$

Mo uses a single bar model to work out: $\frac{1}{5} \times 4 = \frac{4}{5}$

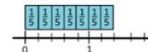


Use this method to work out:

$$\frac{1}{4} \times 3 \qquad 6 \times \frac{1}{8} \qquad \frac{1}{10} \times 8$$

Eva uses a number line and repeated addition to work out

$$\frac{1}{5} \times 7 = \frac{7}{5} = 1\frac{2}{5}$$



Use this method to work out:

$$5 \times \frac{1}{8} \qquad \frac{1}{3} \times 3 \qquad \frac{1}{4} \times 7$$

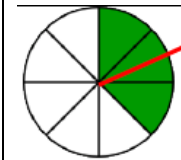
Then, introduce the children to this method, explaining why it works.

$\frac{3}{8}$ of the pizza remains. Two friends are going to share it equally.



Each friend will get half each.

How can we represent this?



Find half of the $\frac{3}{8}$.

Now to ensure you are representing it fairly, you have to make sure all the parts are equal (e.g sixteenths, so draw the rest of the grid out to support in counting that each child has three sixteenths – see below).



Pictorial-

$$1. \quad 3 \times \frac{7}{10} = \frac{21}{10}$$

The reason why this works:

$$2. \quad \frac{3}{1} \times \frac{7}{10} = \frac{21}{10}$$

$\overset{\times 7}{\text{---}}$ $\underset{\times 10}{\text{---}}$

...is because you do have '3 wholes' (or 3/1) – if they are getting too focused on this method alone and start to forget **why** it works, move them back into concrete or abstract methods.

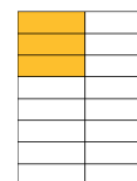
N.B When multiplying mixed fractions, multiply the wholes together first, and then multiply the fraction and the whole together. E.g.

$$\frac{3}{8}$$



Multiply the denominator by the whole number. The numerator stays the same. Why does this work?

$$\frac{3}{8} \div 2$$



This is 3/16 because you have shared 3/8 equally between two people meaning they get half each. Half of 3/8 is 3/16.

$$2\frac{2}{3} \times 4 =$$

$$2 \times 4 = 8$$

$$\frac{2}{3} \times \frac{4}{1} = \frac{8}{3}$$

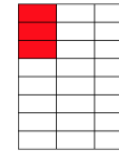
$$8 + \frac{8}{3} = 8\frac{8}{3} = 9\frac{2}{3}$$

Fractions of an amount

Continue with the method used in Year 4.

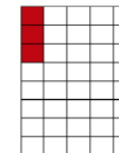
Non-statutory- Pupils should simplify using factors and multiples where appropriate (for example, $\frac{6}{9} = \frac{2}{3}$ or $\frac{1}{4} = \frac{2}{8}$.) Children should use the method of finding the highest common factor of both the numerator and the denominator and divide both by this.

$$\frac{3}{8} \div 3$$



This pattern continues. $\frac{3}{8}$ shared equally between three people will mean they get one third each. One Third of $\frac{3}{8}$ is $\frac{1}{8}$.

$$\frac{3}{8} \div 5$$

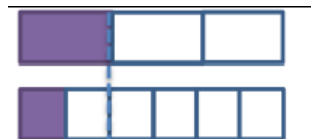


Abstract-

$$\frac{1}{3} \div 2 = \frac{1}{6}$$

Pupils understand the strategy and can calculate mentally.

If necessary, they can use simple drawn models if they confuse their own understanding.



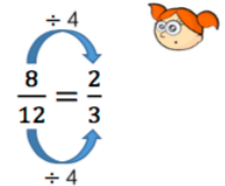
divided by 2

Use common factors to simplify fractions.

Abstract-Children use their understanding of the highest common factor to simplify fractions, building on their knowledge of equivalent fractions in earlier years.

Alex is simplifying $\frac{8}{12}$ by dividing the numerator and denominator by their highest common factor.

Factors of 8: 1, 2, 4, 8
 Factors of 12: 1, 2, 3, 4, 6, 12
 4 is the highest common factor.



Use Alex's method to simplify these fractions:

$$\frac{6}{9} \quad \frac{6}{18} \quad \frac{10}{18} \quad \frac{10}{15} \quad \frac{15}{50}$$

Mo has 3 boxes of chocolates. 2 boxes are full and one box is $\frac{4}{10}$ full.

To simplify $2\frac{4}{10}$, keep the whole number the same and simplify the fraction. $\frac{4}{10}$ simplifies to $\frac{2}{5}$

$$2\frac{4}{10} = 2\frac{2}{5}$$


Use Mo's method to simplify:

$$3\frac{4}{8}, 5\frac{9}{21}, 2\frac{7}{21}, \frac{32}{10}, \frac{32}{6}$$

Compare and order fractions including fractions >1.

Children use their knowledge of equivalent fractions to compare fractions where the denominators are not multiples of the same number. They find the lowest common multiple of the denominators in order to find equivalent fractions with the same denominators. Children then compare the numerators to find the larger or smaller fraction. Encourage children to also use their number sense to visualise the size of the fractions before converting.

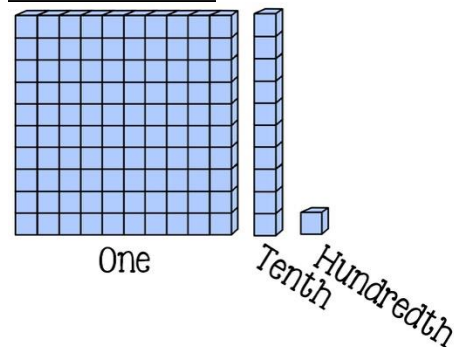
Pictorial-

		<p>Use the bar models to compare $\frac{3}{4}$ and $\frac{2}{3}$</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> <div style="border: 1px solid black; width: 100px; height: 20px; margin-bottom: 5px;"></div> <div style="border: 1px solid black; width: 100px; height: 20px;"></div> </div> <div style="text-align: center;"> <div style="border: 1px solid black; width: 100px; height: 20px; margin-bottom: 5px;"></div> <div style="border: 1px solid black; width: 100px; height: 20px;"></div> </div> </div> <p>_____ is greater than _____</p> <p>_____ is less than _____</p> <p>Abstract-</p> <p>Dora is comparing $\frac{5}{6}$ and $\frac{3}{4}$ by finding the lowest common multiple of the denominators.</p> <div style="display: flex; justify-content: space-between;"> <div> <p>Multiples of 6: 6, 12, 18, 24</p> <p>Multiples of 4: 4, 8, 12, 16,</p> <p>12 is the LCM of 4 and 6</p> </div> <div style="text-align: center;"> $\frac{5}{6} = \frac{10}{12}$ $\frac{3}{4} = \frac{9}{12}$ $\frac{10}{12} > \frac{9}{12}$ </div> <div style="text-align: right;">  </div> </div> <p>Use Dora's method to compare the fractions.</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div>$\frac{4}{5} \bigcirc \frac{3}{4}$</div> <div>$\frac{3}{5} \bigcirc \frac{4}{7}$</div> <div>$\frac{3}{4} \bigcirc \frac{7}{10}$</div> <div>$2\frac{2}{5} \bigcirc 2\frac{3}{8}$</div> </div>
Decimals		
Year 4	Year 5	Year 6
<p>Recognise and write decimal equivalents of any number of tenths or hundredths.</p> <p>Recognise and write decimal equivalents to $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$.</p> <p>Find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as one, tenths and hundredths.</p> <p>Round decimals with one decimal place to the nearest whole number.</p> <p>Compare numbers with the same number of decimal places up to two decimal places.</p>	<p>Read and write decimal numbers as fractions e.g. $0.71 = \frac{71}{100}$.</p> <p>Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents.</p> <p>Round decimals with two decimal places to the nearest whole number and to one decimal place.</p> <p>Read, write, order and compare numbers with up to three decimal places.</p> <p>Multiplying by 10, 100 and 1000.</p> <p>Add and subtract decimals, including a mix of whole numbers and decimals (non-statutory).</p>	<p>Calculate decimal fraction equivalents (e.g. $0.375 = \frac{3}{8}$.)</p> <p>Identify the value of each digit in numbers given to three decimal places.</p> <p>Multiply and divide numbers by 10,100 and 1000 giving answers up to three decimal places.</p> <p>Multiply one-digit numbers with up to two decimal places by whole numbers.</p> <p>Use written division methods where the answer has up to two decimal places.</p>

Recognise and write decimal equivalents of any number of tenths or hundredths.

Tenths as decimals

Concrete- Using the hundred square and base 10, children can recognise the relationship between $1/10$ and 0.1 .

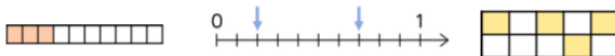


Children read and represent tenths on a place value grid. They see that the tenths column is to the right of the decimal point.

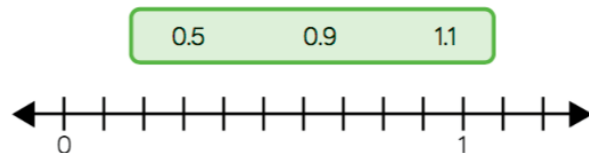


Pictorial-

What fractions and decimals are represented in these diagrams?



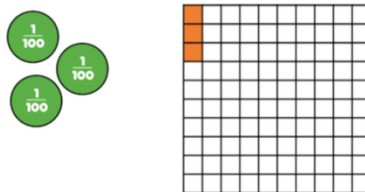
Place the decimals on the number line.



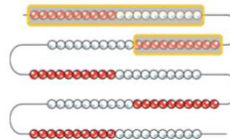
Read and write decimal numbers as fractions e.g. $0.71 = 71/100$.

Concrete/ Pictorial-

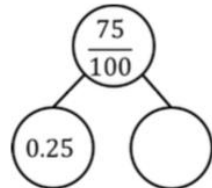
What fraction is shown in both representations?
Can you convert this in to a decimal?



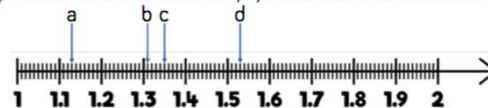
If the whole bead string represents one whole, what decimal is represented by the highlighted part? Can you represent this on a 100 square?



How many different ways can you complete the part-whole model using fractions and decimals?



Write down the value of a, b, c and d as a decimal and a fraction.



Abstract-

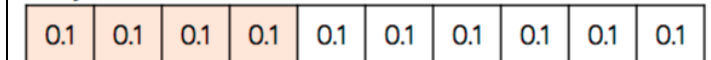
Calculate decimal fraction equivalents (e.g. $0.375 = 3/8$.)

Decimals to fractions:

Concrete/ Pictorial- Children start with a decimal and use their place value knowledge to help them convert it to a fraction.

What decimal is shaded?

Can you write this as a fraction?



Complete the table.

Decimal	Fraction in tenths or hundredths	Simplified fraction
0.6	$\frac{6}{10}$	$\frac{3}{5}$
0.1 0.1 0.1		
0 1		
0.95		

Abstract- Children will convert the fractions to having a denominator as either 10, 100 or 1000 and will then simplify this.

Convert the following decimals to fractions:

0.2=

1.34=

0.324=

Fractions as decimals- method 1

At this point children should know common fractions, such as thirds, quarters, fifths, and eighths as decimals.

Abstract-

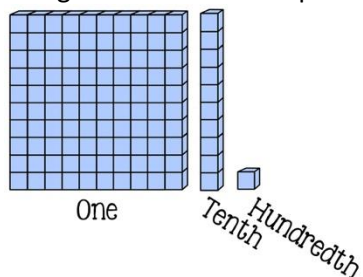
Ones	Tenths	There are <input type="text"/> ones and <input type="text"/> tenths. <input type="text"/> ones + <input type="text"/> tenths = 3 + 0.2 = 3.2
3	2	

Use the place value grid and stem sentences to describe the decimals:

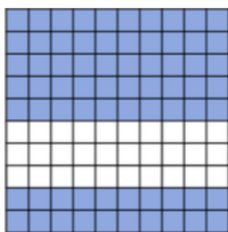
4.0 5.9 2.2

Hundredths:

Using the hundred square and Base 10, children can recognise the relationship between 1/100 and 0.01.



Alex and Eva have been asked to write the decimal shaded on the 100 grid.



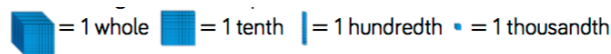
Children read and represent hundredths on a place value grid.

Decimal	Decimal - expanded form	Fraction	Fraction - expanded form	In words
3.24	$3 + 0.2 + 0.04$	$3 \frac{24}{100}$	$3 + \frac{2}{10} + \frac{4}{100}$	Three ones, two tenths and four hundredths.
3.01		$3 \frac{1}{100}$		
			$3 + \frac{4}{10} + \frac{2}{100}$	
				Two ones, three tenths and two hundredths.

Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents.

Children build on previous learning of tenths and hundredths and apply this to understanding thousandths.

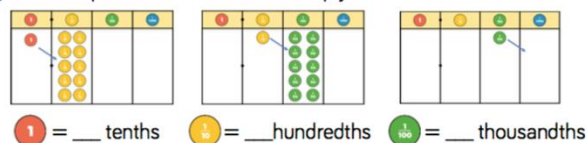
Concrete-



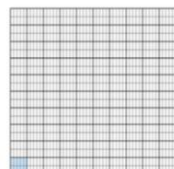
Use Base 10 to build:

- 4 wholes, 4 tenths, 4 hundredths, 4 thousandths
- 5 tenths, 7 hundredths and 5 thousandths
- 2.357

Use the place value counters to help you fill in the final chart.



Pictorial-



What has this hundred square been divided up into?
How many thousandths are there in one hundredth?
How many thousandths are in one tenth?

Children explore how finding an equivalent fraction where the denominator is 10, 100 or 1000 makes it easier to convert from a fraction to a decimal.

Abstract-

Match the fractions to the equivalent decimals.

$\frac{2}{5}$	0.04
$\frac{1}{25}$	0.4
$\frac{1}{4}$	0.25

Use your knowledge of known fractions to convert the fractions to decimals. Show your method for each one.

$\frac{7}{20}$	$\frac{3}{4}$	$\frac{2}{5}$	$\frac{6}{200}$
----------------	---------------	---------------	-----------------

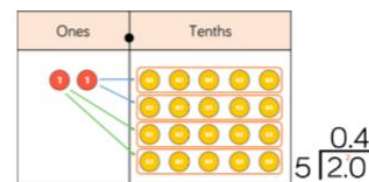
Fractions as decimals- method 2

It is important that children recognise that $\frac{3}{4}$ is the same as 3 divided by 4. They can use this understanding to find fractions as decimals by then dividing the numerator by the denominator.

Concrete/Pictorial-

In the example provided, we cannot make any equal groups of 5 in the ones column, so we have exchanged the 2 ones for 20 tenths. Then we can divide 20 into groups of 5.

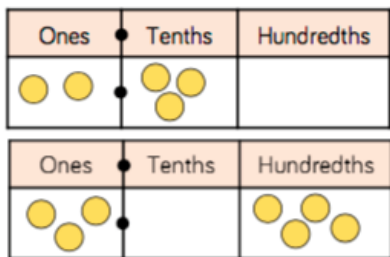
Deena has used place value counters to write $\frac{2}{5}$ as a decimal. She has divided the numerator by the denominator.



Use this method to convert the fractions to decimals. Give your answers to 2 decimal places.

$\frac{1}{2}$	$\frac{3}{4}$
---------------	---------------

Abstract-



Pictorial-

Represent the decimals on a place value grid and in a part whole model.
How many ways can you partition each number?

0.27 0.72 0.62



Abstract

Complete the table.

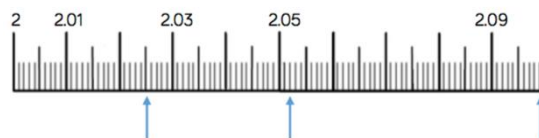
Image	Words	Fraction	Decimals
	56 hundredths		
		$\frac{17}{100}$	
			0.2

Recognise and write decimal equivalents to $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$.

Concrete/Pictorial/Abstract-

Children will use concrete and pictorial representations to support the conversion.

The arrows are pointing to different numbers.
Write each number as a decimal and then as a mixed number.



Abstract-

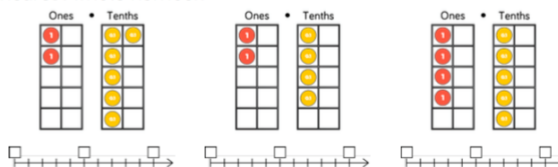
Write 243/1000 in three different ways.

Round decimals with two decimal places to the nearest whole number and to one decimal place.

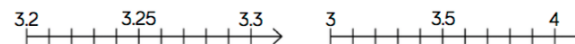
Concrete/Pictorial-

Number lines support children to understand where numbers appear in relation to other numbers and are important in developing conceptual understanding of rounding.

Complete the number lines and round the representations to the nearest whole number:



Use the number lines to round 3.24 to the nearest tenth and the nearest whole number.



Abstract-

Round the following numbers to the nearest whole number and the nearest tenth:

-0.72

-2.56

-7.99

$$\frac{3}{5} = 0.6$$

$$\frac{3}{8} = 0.375$$

When there is a remainder, the children will continue to add zeros, (as in the example above) unless it's taking it to more than 3 decimal places.

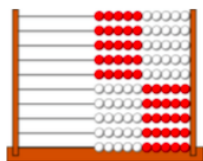
Multiply and divide numbers by 10,100 and 1000 giving answers up to three decimal places.

This will involve the same methods as in Year 5, however will involve numbers with up to three decimal places.

Multiply one-digit numbers with up to two decimal places by whole numbers.

Concrete- Children use concrete resources to multiply decimals and explore what happens when you exchange with decimals.

Here is a rekenrek with 100 beads.



___ out of 100 beads are red.
___ out of 100 beads are white.
 $\frac{\square}{100}$ are red, and $\frac{\square}{100}$ are white.

Half of the beads are red, and half of the beads are white.

$$\frac{1}{2} = \frac{50}{100} = \frac{5}{10}, \text{ so } \frac{1}{2} \text{ is } \underline{\quad\quad} \text{ as a decimal.}$$

Find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as one, tenths and hundredths.

Children need to understand when dividing by 10 the number is being split into 10 equal parts and is becoming 10 times smaller. Children will use the method of moving the numbers (counters) one column to the right.

Concrete-

Teddy uses counters to make a 2-digit number.

Tens	Ones	Tenths	Hundredths
●	●●		

To divide the number by 10, we move the counters one column to the right.

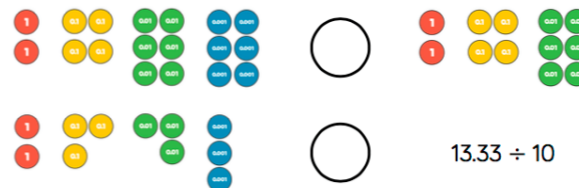
What is the value of the counters now?

Pictorial/ Abstract- Children replace the counters with digits. It is worth noting here that the decimal point does not have its own column.

Read, write, order and compare numbers with up to three decimal places.

Concrete- Children can use place value counters to represent the numbers they are comparing.

Use $<$, $>$ or $=$ to make the statements correct.



Pictorial- Number lines support children to understand where numbers appear in relation to other numbers.

Place the numbers in ascending order on the number line.



Abstract-

Place in descending order.

- 0.123 0.321 0.231 0.103
- 3.2 km 3.21 km 3.212 km 3202 m
- 65.394 65.309 63.999 65.493

Multiplying by 10, 100 and 1000.

Concrete- Children will look at moving the counters in a place value grid to the left in order to multiply by multiples of 10 and move it to the right to divide. Children may have previously made the generalisation that when a number is ten times greater they put a zero on the end of the original number.

Use the place value counters to multiply 1.212 by 3

Complete the calculation alongside the concrete representation.

Tens	Ones	Tenths	Hundredths	Thousandths
	1	2	2	
	1	2	2	
	1	2	2	

Pictorial-

A jar of sweets weighs 1.213 kg.

How much would 4 jars weigh?



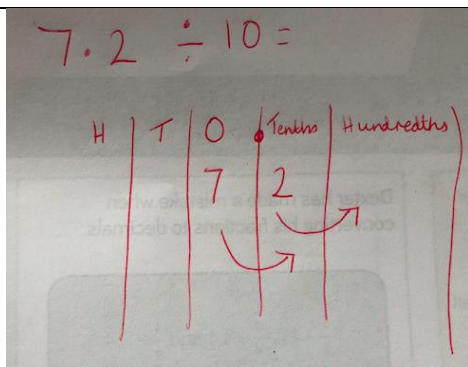
Abstract-

$$\begin{array}{r} 3.12 \\ \times 4 \\ \hline 12.48 \end{array}$$

Use written division methods where the answer has up to two decimal places.

Concrete- Children continue to use concrete resources to divide decimals and explore what happens when exchanges take place. Children build on their prior knowledge of sharing and grouping when dividing and apply this skill in context.

Use place value counters and group e.g. 3.69 divided by 3



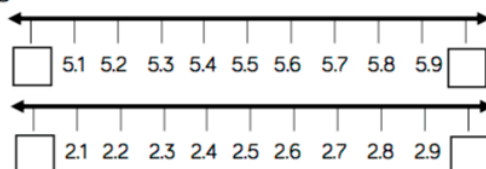
The same method would be used for dividing digits by 100, however this time moving the digits 2 places to the right.

Round decimals with one decimal place to the nearest whole number.

Children will look at the digits in the tenths column to understand whether to round a number up or not. Children often struggle with knowing which whole number is either side of the decimal. Children need to be taught that if a number is exactly half-way, then by convention we round up to the next integer.

Pictorial-

Which integers do the decimals lie between?



Use the place value grid to multiply 3.24 by 10, 100 and 1,000

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths
			3 2 4	2	4

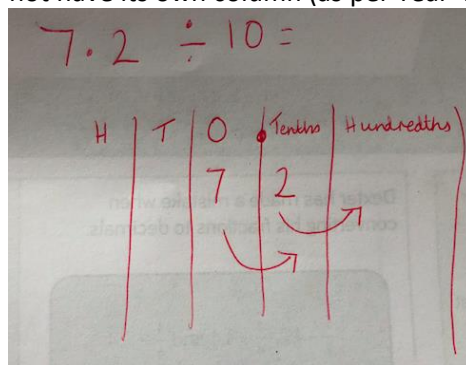
When you multiply by ____, you move the counters ____ places to the left.

Use the place value grid to divide 14.4 by 10, 100 and 1,000

T	O	Tths	Hths	Thths	TThth
1	4	4			

When you divide by ____, you move the counters ____ places to the right.

Pictorial- Children replace the counters with digits. It is worth noting here that the decimal point does not have its own column (as per Year 4).



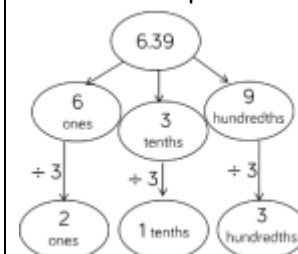
Abstract-

Complete the table below.

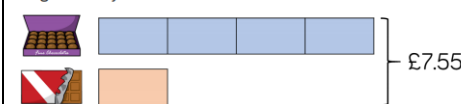
	$\times 10$	$\times 100$	$\times 1,000$
3.14			
13			
0.233			

Ones	Tenths	Hundredths
1 1 1	0.1 0.1 0.1	0.01 0.01 0.01

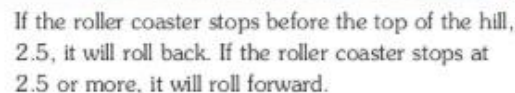
Pictorial- Use part-whole and bar models



A box of chocolates costs 4 times as much as a chocolate bar. Together they cost £7.55



Abstract- Short division to divide decimals by an integer.



Circle the numbers that round up to the nearest whole number.

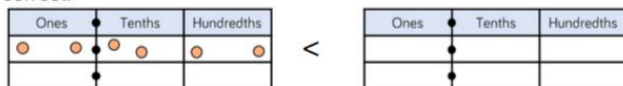
4.5 3.7 2.3 4.2 16.8 1.9

Children apply their understanding of place value to compare numbers with decimals with up to two decimal places. Emphasise the importance of 0 as a place holder when making a comparison. **Remember- using money can be really useful here.**

Write the numbers shown and compare using $<$ or $>$



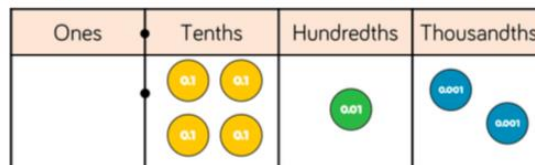
Draw counters in the place value chart to make the statement correct.



Abstract-

Concrete- Children will use place value counters and place value charts to support adding decimals and understand what happens when we exchange between columns.

Use this place value chart to help answer the questions.



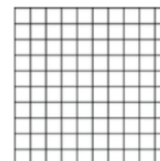
- What number is one hundredth more?
- Add 0.3, what number do you have now?
- How many more thousandths can I add before the hundredths digit changes?

Here is a number.



- What is three tenths less than the number?
- Take away 0.02, what is your number now?
- Subtract 5 thousandths. What is the final number?

Each box in this hundred square represents one hundredth of the whole. Use this to answer:



$$0.07 + 0.78$$

$0.87 + 0.07$

Find the difference between the two numbers using the number line.

0.424

0.618

Complete.

5.5	<input type="radio"/>	5.7	0.37	<	0.7
0.14	<input type="radio"/>	0.29	2.22	>	2.2
1	<input type="radio"/>	0.64	1.1	>	1.1
3.32	<input type="radio"/>	3.23	9.9	<	9.9

Abstract-

Use the column method to complete the additions.

$$0.45 + 0.5$$

$$0.45 + 0.05$$

$$0.45 + 0.005$$

Calculate.

$$\begin{aligned} 0.584 - 0.154 &= \\ 0.684 - 0.254 &= \\ 0.685 - 0.255 &= \end{aligned}$$

$$\begin{aligned} 0.44 - 0.1 &= \\ 0.44 - 0.09 &= \\ 0.44 - 0.11 &= \end{aligned}$$

Complements to 1

Children will find decimals which sum to make 1. It is important for children to see the links with number bonds to 10, 100 and 1000.

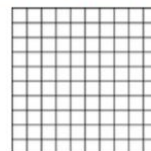
Pictorial-

Using a blank hundred square, where each square represents one hundredth, find the complements to 1 for these numbers.

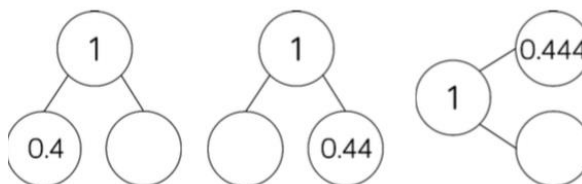
$$0.55 + \boxed{} = 1$$

$$1 = 0.32 + \boxed{}$$

$$0.11 + 0.5 + \boxed{} = 1$$



Complete the part-whole models.



Use the number line to find the complements to 1

0.324 _____ 1

0.459 _____ 1

Abstract-

$$0.333 + \boxed{} = 1$$

Adding- crossing the whole

Concrete-

Use the place value grid to answer $0.453 + 0.664$

Ones	Tenths	Hundredths	Thousandths
	0.1 0.1	0.01 0.01	0.001 0.001
	0.1 0.1	0.01 0.01	0.001 0.001
	0.1 0.1 0.1	0.01 0.01 0.01	0.001 0.001

Pictorial-

Amir is using complements to 1 to add decimals.

$$\begin{array}{r} 0.45 \\ + 0.67 \\ \hline 0.55 + 0.12 \end{array}$$

$$0.45 + 0.55 + 0.12 = 1.12$$

Use Amir's method
to solve:

- a) $0.56 + 0.78$
- b) $3.42 + 0.79$

Abstract-

Use the column method to solve the additions.

$$0.47 + 0.6$$

$$0.982 + 0.18$$

$$0.92 + 0.8$$

Adding and subtracting decimals with the same and different amount of decimal places

Where children are adding and subtracting decimals with different amount of decimal places, encourage them to use zeros as place holders.

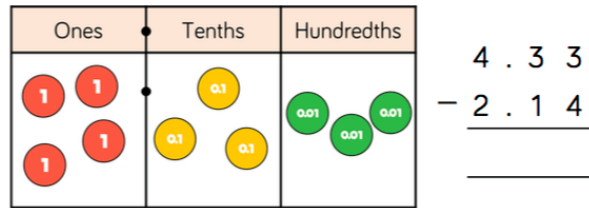
Concrete-

Use the place value chart to add 3.45 and 4.14

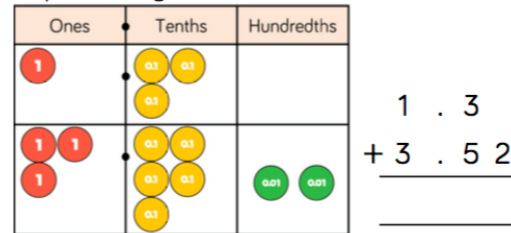
Ones	Tenths	Hundredths
1 1	0.1 0.1	0.01 0.01
1	0.1 0.1	0.01 0.01
1 1	0.1	0.01 0.01
1 1		0.01 0.01

$$\begin{array}{r} 3.45 \\ + 4.14 \\ \hline \end{array}$$

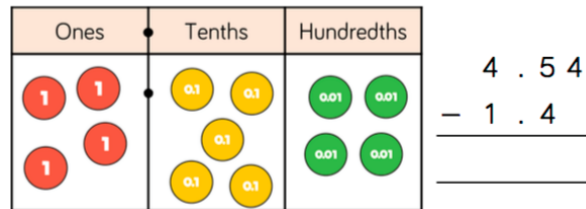
Use the place value chart to find the to answer $4.33 - 2.14$



Use the place value grid to add 1.3 and 3.52



Use the place value grid to help subtract 1.4 from 4.54



Abstract-

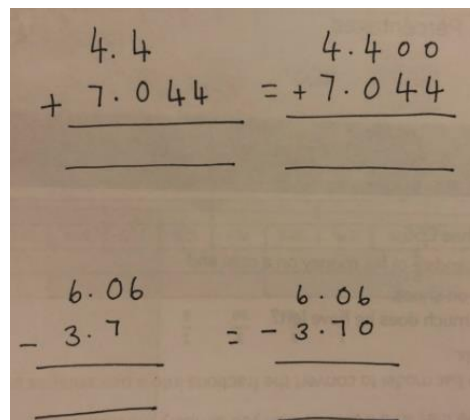
Use the column method to solve these additions.

$$\begin{array}{r} 4.42 \\ + 7.63 \\ \hline \end{array}$$

$$\begin{array}{r} 4.55 \\ + 3.07 \\ \hline \end{array}$$

Use the column method to answer these questions.

$$\begin{array}{r} 6.4 \\ -3.8 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 5.05 \\ -2.15 \\ \hline \\ \hline \end{array}$$



Handwritten examples of column methods:

$$\begin{array}{r} 4.4 \\ + 7.044 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 4.400 \\ + 7.044 \\ \hline \\ \hline \end{array}$$

$$\begin{array}{r} 6.06 \\ - 3.7 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 6.06 \\ - 3.70 \\ \hline \\ \hline \end{array}$$

Percentages

Year 5

Recognise that the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred'.
Write percentages as a fraction with denominator 100, and as a decimal.
Solve problems which require knowing percentage and decimal equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{5}$ and those fractions with a denominator of a multiple of 10 or 25.

Year 6

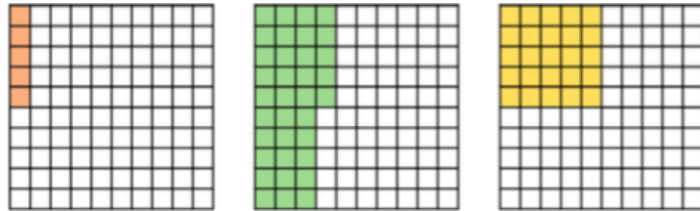
Recall and use equivalences between simple fractions, decimals and percentages, including in different contexts.
Solve problems involving the calculation of percentages (e.g. of measures, and such as 15% of 360) and use of percentages for comparison.

Recognise that the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred'.

Children are introduced to 'per cent' for the first time and will understand that 'per cent' relates to 'number of parts per hundred'.

Pictorial-

Complete the sentence stem for each diagram.

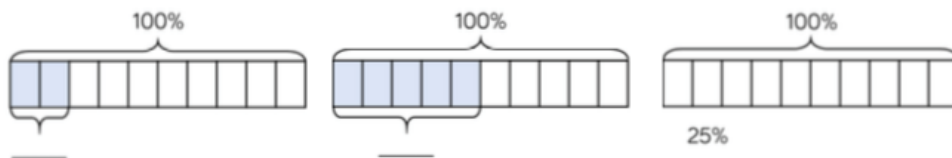


There are ____ parts per hundred shaded. This is ____%

Complete the table.

Pictorial	Parts per hundred	Percentage
	There are 51 parts per hundred.	
		75%

Complete the bar models.



Abstract-

$$56/100 = \underline{\hspace{2cm}} \%$$

$$99/100 = \underline{\hspace{2cm}} \%$$

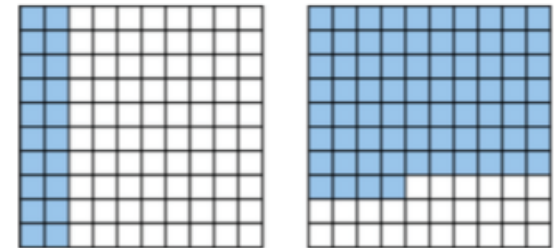
Recall and use equivalences between simple fractions, decimals and percentages, including in different contexts.

Fractions to percentages

It is important that children understand that 'percent' means 'out of 100'. Children will convert fractions to equivalent fractions where the denominator is 100 in order to find the percentage equivalent.

Pictorial-

What fraction of each hundred square is shaded?
Write the fractions as percentages.



Abstract-

Complete the table.

Fraction	Percentage
$\frac{1}{2}$	
$\frac{1}{4}$	
$\frac{1}{10}$	
$\frac{1}{5}$	

Fill in the missing numbers.

$$\frac{12}{100} = \boxed{\hspace{1cm}} \%$$

$$\frac{\boxed{\hspace{1cm}}}{100} = 35\%$$

$$\frac{12}{50} = \frac{\boxed{\hspace{1cm}}}{100} = \boxed{\hspace{1cm}} \%$$

$$\frac{44}{\boxed{\hspace{1cm}}} = \frac{22}{100} = 22\%$$

$$32/100 = \underline{\hspace{2cm}} \%$$

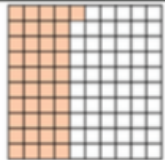
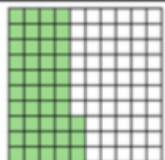
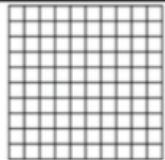
$$4/100 = \underline{\hspace{2cm}} \%$$

Write percentages as a fraction with denominator 100, and as a decimal.

Children represent percentages as fractions using the denominator 100 and make the connections to decimals and hundredths. Children will recognise percentages, decimals and fractions are different ways of expressing proportions.

Pictorial-

Complete the table.

Pictorial	Percentage	Fraction	Decimal
	41 parts per hundred 41%	41 out of 100 $\frac{41}{100}$	41 hundredths 0.41
			
	7 parts per hundred 7%		

Abstract-

Record the fractions as decimals and percentages.

$$\frac{120}{300} \quad \frac{320}{400} \quad \frac{20}{200} \quad \frac{12}{50}$$

Solve problems which require knowing percentage and decimal equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{5}$ and those fractions with a denominator of a multiple of 10 or 25.

When children are secure with the percentage and decimal equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{5}$, they then consider denominators of a multiple of 10 or 25.

Concrete-

Equivalent fractions, decimals and percentages

A common misconception is that 0.1 is equivalent to 1%. Use diagrams from decimals work to support understanding the difference between tenths and hundredths and their equivalent percentages.

Abstract-

Complete the table.

Decimal	Fraction	Percentage
0.35	$\frac{35}{100}$	35%
0.27		
0.6		
0.06		

Use <, > or = to complete the statements.

$$0.36 \bigcirc 40\% \qquad \frac{7}{10} \bigcirc 0.07$$

$$0.4 \bigcirc 25\% \qquad 0.4 \bigcirc \frac{1}{4}$$

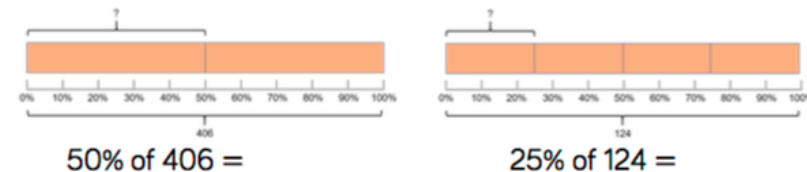
Solve problems involving the calculation of percentages (e.g. of measures, and such as 15% of 360) and use of percentages for comparison.

Method 1

Children use known fractional equivalences to find percentages of amounts.

Pictorial- Bar models and other visual representations may be useful in supporting this e.g. $25\% = \frac{1}{4}$ so we divide into 4 equal parts. In this step, we focus on 50%, 25%, 10% and 1% only.

Use the bar models to help you complete the calculations.



Abstract-

Use a bead string to show me:

0.25 0.3 0.2 0.5

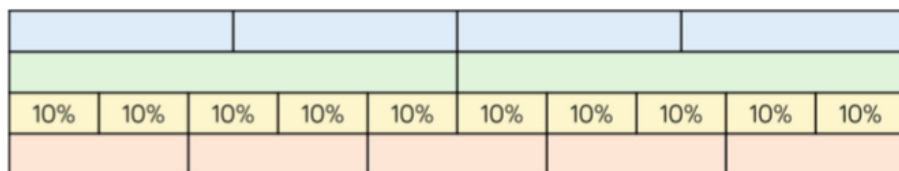
What are these decimals as a percentage?

What are they as a fraction? Can you simplify the fraction?

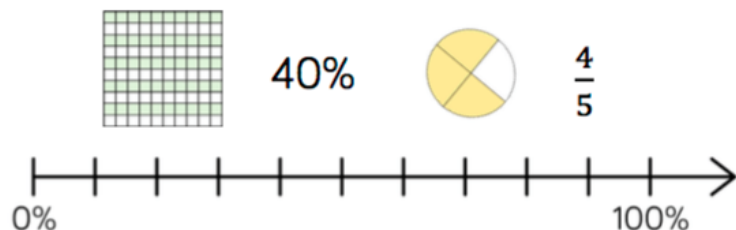
Pictorial-

Use the bar model to convert the fractions into a percentages and decimals.

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{10}$ $\frac{1}{5}$



Draw arrows to show the position of each representation on the number line.



Abstract-

Find:

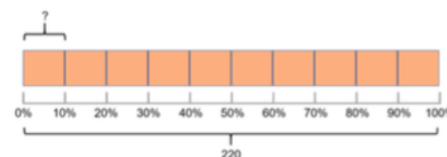
50% of 300	25% of 300	10% of 300	1% of 300
50% of 30	25% of 30	10% of 30	1% of 30
50% of 60	25% of 60	10% of 60	1% of 60

Method 2

Children build on the last step by finding multiples of 10% and other known percentages. They explore different methods of finding certain percentages e.g. Finding 20% by dividing by 10 and multiplying by 2. They also explore finding 5% by finding half of 10%. Using these methods, children build up to find percentages such as 35%.

Pictorial-

Mo uses a bar model to find 30% of 220



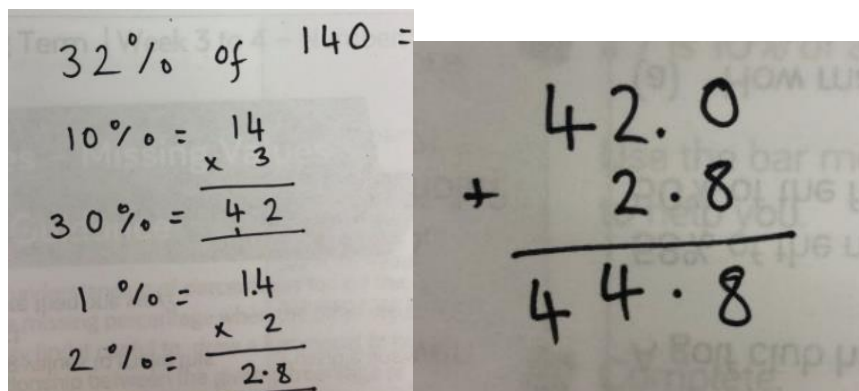
10% of 220 = 22, so 30% of 220 = $3 \times 22 = 66$

Abstract-

Calculate:

15% of 60 m 35% of 300 g 65% of £20

Children will further this by finding any percentage of an amount e.g. 32% of 140.



Sort the fractions, decimals and percentages into the correct column.

50%

100%

$\frac{30}{60}$

Seven tenths

60%

0.25

70 hundredths

$\frac{1}{4}$

7%

Less than $\frac{1}{2}$	Equal to $\frac{1}{2}$	Greater than $\frac{1}{2}$

Which is closer to 100%, $\frac{4}{5}$ or 50%? How do you know?